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UNITY OF PURPOSE,

OR

RATIONAL ANALYSIS:

BEING AN EXPOSITION

OF THE

QUADRATURE OF THE CIRCLE.

AND

THE LAW OF GRAVITY.

BY AUGUSTUS YOUNG.

) are not, perhaps, very attractive speculations; they disturb old
ite associations; they serve to reduce many cherished traditions,
nfully acquired knowledge, to obsolete lore; but these things are
we must accustom ourselves to regard them and their consequences
without shrinking."

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OF THE

Quadrature of the Circle.

Preliminary Remarks.

In attempting an exposition of the Quadrature, it is my purpose to rely wholly upon an application of such principles in regard to polygonal economy (together with their legitimate corrolories,) as have long since been well demonstrated and universally acknowledged and assented to as indubitable geometric truths.

It is not, therefore, my purpose to demonstrate anew any of those principles; but it is my intention to make no allegation, the truth of which may not be readily deduced from demonstrated principles.

These remarks however, are only introduced to apply to such principles and corrolories as are incident to a direct exposition of the Quadrature, amongst which are included certain necessary conceptions in regard to the Circle; as that the periphery of the circle, for example, is, in regard to numerical magnitude, a mean proportional between the perimeters of its ultimate inscribed and circumscribed polygons; and consequently, that the circle when sought through an infinite series of polygons, is itself conceived to be a polygon; and consequently, that its measures are conceived to be polygonal, notwithstanding its measures may in fact be reduced to its diameter and circumference only,—being wholly divested of sides and angles.

But in regard to such speculations as refer exclusively to Le-

genders process for deducing the Quadrature, together with such proposed amendments to his process as may be offered, it is not here contended that all the propositions or positions put forth in regard to the same, rest upon indubitable demonstration or conclusion; for it must be confessed, that rectilinear geometry, so far as its principles have as yet been disclosed, has not been over bountiful in furnishing direct evidence either for the support or overthrow of Legenders process; with these remarks therefore, I will proceed to the investigation:

SECTION FIRST.

Regular polygons are the only polygons here treated of; and all polygons having the same number of sides are called *like* polygons.

The *diameter* of a polygon is the diameter of its inscribed circle; the *diagonal* of a polygon is the diameter of its circumscribed circle; and the *quadrant* of a polygon is one fourth of its circumference.

A *major* polygon is any polygon whose diameter is 1; a *minor* polygon is any polygon whose diagonal is 1; and a *meter* polygon is any polygon whose area is ,5.

A *triplet*, consists of a like *major*, *minor* and *meter* polygon, taken in connection with each other; and the number of sides of either polygon of a triplet is taken as the number of sides of the triplet.

A *series* commences with any given triplet, and thence progresses towards the circle by a constant doubling of the number of sides of the successive triplets.

A *duple* consists of any two consecutive triplets of a series.

The two last triplets of a series are called the *focal* and *final* triplets, and constitute the final duple of the series.

Any triplet of more sides than the square, is called a *minor* triplet; and any triplet of the series commencing with the octagon, is called a prime triplet; and any duple consisting of prime triplets is called a prime duple.

The cube root of ,5—or, 7937005 &c, is denoted by the letter P.

When P is a mean between any two notations, they are said to

be *adverse*—or either is the adverse of the other; and in such case, the reciprocal of either divided by twice the sum of the other, gives P.

When P is the farther of two means from one notation to another, the square of the nearer is half the reciprocal of the farther; and the farther is half the square of the reciprocal of the nearer; and the reciprocal of either divided by twice the sum of the other gives the nearer: Hence when two notations coincide in P, the square of either is half the reciprocal of the other.

The notations of any triplet when such triplet is considered either in the abstract, or as the first triplet of a duple, are expressed by large capital letters as follows:

A denotes the area of a minor polygsn.

M denotes the quadrant or area of a major polygon.

N denotes a mean between A and M, and is equal A of the triplet of twice the sides.

B denotes the diagonal of a meter polygon.

D denotes the diameter of a meter polygon.

C denotes a mean between B and D, and is equal B of the triplet of twice the sides.

R denotes a mean between N and M, and is consequently the farther of three means from A to M.

Z denotes half the square of the reciprocal of N.

When both triplets of a duple are considered in connection with each other, the notations of the first triplet are expressed by large capital letters, and the notations of the second triplet by small capital letters.

For the reason that the quadrant of any given polygon of whatever dimensions, is a mean between the area of such polygon and M of a *like* polygon, hence N of any triplet is equal tne quadrant of the minor polygon of such triplet.

For the reason that a mean between the diagonal and diameter of a polygon of any given area, is equal the diagonal of the polygon of twice the sides of equal area; hence C of any triplet is equal B of the triplet of twice the sides, of equal area; hence in progressing through any series, N and C are successively converted into A and B.

In any triplet of any series, P is the farther of two means from A to B, from N to C, from Z to M, from M to D, and from R to a mean between C and D; hence R in P is as well a mean between C and D, as between N. and M.

Thus N and C of any triplet denote the numerical positions of A and B of the triplet of twice the sides, or A and B of any prime triplet denote the numerical positions of N and C of the triplet of half the sides.

In any triplet of any series, N is to D as R is to C, or as M is to B, or B to Z: Hence D is to R as C is to M.

D of any triplet is a mean between A and Z: Hence D of any duple is a mean between N and Z.

C of any triplet is as well a mean between Z and N, as between B and D, and is the farther of three means from Z to P.

A mean between N and B is a mean between M and D, and also between R and C, and is the farther of three means from M to P.

A mean between N and C is a mean between D and R, and also between N and D, and is the farther of three means from N to P

By any system, R progresses into P; and by Legender, it progresses much farther.

By any system, M and D progress to a coincidence with each other in P; and by Legender, they progress much farther.

By no system do N and C progress as far as to a coincidence in P, however near the approach: for if we conceive N or C to be in P, we necessarily conceive M to be below N, and D above C.

By any system, D and N are conceived to progress to a coincidence with each other; M and Z are conceived to progrees to a coincidence with each other; and R and D are conceived to progress to a coincidence with each other, forming the junctives Dn, Rd and Mz; and by any system those three junctives occur in one and the same duple.

For P is as well the farther of two means from Z to N, as from M to D: And R is a mean between N and M, and D is a mean between N and Z: Hence Rd is a mean between Mz and N, and is in the same duple with Dn and Mz.

By any system, R of the focal triplet is equal M; for R of the

focal triplet is the utmost mean that can actually exist between N and M, and is consequently equal M of the final triplet or circle.

Hence when R becomes equal M; that is, when M is a mean between M and N, the progression is ended by any system.

And such is assumed by Legender in his logarithmic process, however erroneous his process may be in fact; as will be seen from a correct translation of his process into the language of this article.

Nevertheless, Legenders process gives M truly when applied to the trigon, or triplet in which N is above D, R above C, M above B and B above Z.

So also Legender's process gives M truly when applied to the square, or polygon of perfect rectitude, in which N is equal D, R equal C, and M equal B or Z.

But the proposition is here suggested, that his process will not give M truly when applied to any triplet of more sides than the square, or in which N is below D, R below C, M below B and B below Z.

And it may be here remarked that any process similar to Legenders, that will give M truly when applied to any other triplet than the square, will give M truly when applied to the square; but the reverse of this will not hold.

But I will defer for the present, any farther notice of Legenders logarithmic process, and will proceed to treat the subject by way of rational analysis.

SECTION SECOND.

Let x denote the difference between N and D, or R and C, or M and B, or B and Z of any *prime* triplet; and let twice x &c. imply twice the proportional value of x &c.: Hence our exposition by the use of x, is confined wholly to the prime triplets and duples.

It is here assumed that the value of x is greater in the octagon than in any other prime triplet; and that from the octagon onward it continually decreases until it becomes less than an assignable value in the final triplet.

Whereas Legender assumes that the value of x constantly increases from the octagon onward; and consequently, that x is at its maximum value in the final triplet.

But whether the value of x increases or decreases from the octagon onward, the value of x is never conceived to be wholly extinguished; hence even in the final triplet, x is still conceived to exist, notwithstanding it is then of no actual value.

By any system, in the final duple (but in no other,) the *proportional* value of x is the same between like notations of the first and second triplets of the duple: Thus in the final duple N is to D as N is to D, or N is to N as D is to D; so also Z is to M as z is to M, or Z is to z as M is to M.

As D is to R as C is to M, hence when the progression has proceeded until D is x below R, and C x below M, the notations of such triplet from N to Z, are in continued proportion; and in a formula of the same, x will denote the difference between any two consecutive terms of the formula, thus, N x D x R x C x M x B x Z.

Thus in such triplet N and M are adverse, and D and C are adverse, and R is the farther of two means from Z to N, from N to C, and from M to D; It is therefore manifest that R of such triplet is in P; and it is also manifest, that N and C of such triplet occupy the numerical positions of A and B of the triplet of twice the sides.

When the progression has proceeded until D and M coincide in P, either D or M may be used to denote the numerical position of both; in which case such notations of the triplet as are in continued proportion, and in which x denotes the difference between any two consecutive terms, will be expressed in a formula thus, A x N x D x B x Z, or thus A x N x M x B x Z.

Thus D and M of such triplet are in P; for in such triplet, D or M is the farther of two means from A to B, and from Z to N; and in such triplet A and Z are adverse, and N and B are adverse; Z is x above B, and B is x above M; D of such triplet is equal M and D is x above N, and N is a mean between A and M; and A and B of such triplet are equal N and C of the triplet of half the sides.

If the triplet whose D and M are in P, is the triplet of twice the sides of the triplet whose R is in P, then a joinder of the two formulas is thus, N x D_N x R_D x C x M_Z x B x Z.

In which case, R in P is equal either D or M; and consequently, by any system, such would be a formula of the final duple, in which the proportional value of x is the same between like notations of the first and second triplets.

It is manifest that R in P, cannot be equal either D or M, without being equal the other also.

We have already seen that R in P, is x below C and twice x above N; and also that D or M in P is x below C and twice x above N.

Consequently, that either R, D or M in P is four times x below Z, twice x below M, and x above D; and that D, or M, is then x above N and twice x below Z; consequently, that R in P is x above N, and twice x below Z; and that either R, D or M in P (but not otherwise,) is thus situated.

Thus D in P (but not otherwise,) is x below C, and is consequently, four times x below Z, for the reason that C is always the farther of three means from Z to P; and D in P is twice x below M, for the reason that M of any prime triplet is twice x below Z; and as D in P is twice x below Z, hence M_Z occurs when D is in P; and R_D occurs in the same duple with M_Z and D_N, and is a mean between M_Z and N: Hence R_D is in P.

So also M in P (but not otherwise,) is twice x above N, and is consequently x above N; for N is a mean between N and M; and as M in P is twice x above N, it is therefore four times x below Z, and twice x below M, and is therefore x above D: Hence M in P is twice x above N, is x above D_N, is twice x below M_Z, and four times x below Z: And R in P is twice x above N, x above D, twice x below M, and four times x below Z.

Hence the junctives occur when M is in P, or twice x above N, and consequently, when M is equal R; or in other words, the junctive R_M occurs in P; or M in P, (but not otherwise,) is a mean between N and M; or thus: M in P (but not otherwise) is twice x above N, and consequently, x above N; and when D_N occurs, M is twice x above N and x above D_N; for D_N is x above N be-

cause D is x above N, and N is a mean between N and M: hence DN occurs when M is in P, and P is the farther of two means from Mz to DN: Hence when the junctives occur, M and D are in P, and R is then equal D.

Hence the coincidence of R and M, (which ends the progression by any system,) is in P; or M in P (but not otherwise,) is a mean between M and N, and is therefore M of the final triplet or circle.

Thus M in P, notwithstanding it cannot be definitely expressed by numbers short of its third power or cube, is nevertheless, the actual quadrant of the circle whose diameter is 1; and D in P is the actual diameter of the circle whose area is ,5.

Such therefore, is an actual determination of the ratio of the quadrant of the circle to the diameter; for P is the farther of two means from ,5 to 1, notwithstanding P cannot have a definite numerical expression, either in its first or second power.

Hence the cube of the diameter of the circle (of whatever dimensions,) is twice the cube of the quadrant; and the cube of the area of a square is twice the cube of the area of its inscribed circle, and one fourth the cube of the area of its circumscribed circle.

Hence the cube of the diagonal of a square of any given area is twice the cube of the diameter of a circle of equal area.

SECTION THIRD.

Legendre's erroneous process, which places M of the circle in ,785398, &c., necessarily places D of the circle in ,797884, &c.; for by the law of progression, P is the farther of two means from M to D of any given triplet, and must be so in respect to the circle, unless M and D of the circle actually coincide in P.

Legendre's process, then, when translated into the language of this article, is this: "The product of M by N of any triplet of any series, divided by half the sum of M and N gives M." Or thus: "The product of N by B, or of M by D of any triplet of any series divided by half the sum of D and B gives M." For these processes applied to any given triplet give the same quotient; and either process when applied to the trigon or square, gives M truly;

but when applied to any other triplet, viz. to any triplet in which N is less than D, M less than B, &c., it gives less than M.

Now any similar process to those of Legendre will give M truly when applied to the square; and any similar process that will give M truly when applied to any other triplet than the square, will give M truly when applied to the square; but the reverse of this will not hold; for the product of D by M of the square, divided by half the sum of D and M, gives M; but such process applied to the trigon gives less than M. And the inference is well sustained by Legendre, that any similar process that will give M truly when applied to the octagon, will give M truly when applied to any triplet below the square, or in which N is below D, M below B., &c.

Thus Legendre's process, which may be denominated the process by M and N, gives M truly when applied to the trigon, or triplet in which N is above D, M above B, &c.; or when applied to the square in which N is equal D, &c.; and it is here assumed that in any triplet of more sides than the square, that is, in any triplet in which N is less than D, M less than B, &c., as well as in the square, the product of D by M divided by half the sum of D and M, gives M truly; which process may be called the process by D and M.

It is here assumed that in the process by M and N, applied to any prime triplet, the divisor is too great for the dividend, and consequently places M too low.

By such process, the value of X constantly increases from the octagon onward to the final duple; and by such process R of any prime duple is above M until R becomes equal M in the final duple.

By such process, the difference between N and D of any prime duple is less than between N and D, and the difference between M and B is less than between M and C, until the difference becomes proportionally equal in the final duple.

Finally, by such process we do nothing but grope; and only conclude that we have arrived at the final goal when we can grope no farther.

But in the process by D and M, X has its maximum value in the

octagon, and its minimum value in the final duple, in which it may be said to be less than an assignable value, and therefore may be said to have no value whatever; consequently, by such process the value of x constantly decreases from the octagon onward.

By such process, R of any prime duple is below M until they become equal in the final duple; and consequently the difference between N and D of any prime duple is greater than between N and D, and the difference between M and B is greater than between M and B, until the proportional difference becomes the same in the final duple.

By such process, the quadrant of the circle whose diameter is 1, is the reciprocal of the diagonal of its equal square; the diagonal of the equal square of any major polygon being twice the product of M by D, or the square root of twice M, viz. twice the quadrant of the meter polygon of such triplet, which is of course the reciprocal of D.

By such process, we cannot progress beyond the coincidence of M and D in P, for the reason that when M and D are in P, the product of M by D divided by half the sum of M and D gives P, and consequently we make no progress. Hence by such process, we have the numerical position of the final goal in view on our departure from the square, or polygon of perfect rectitude.

By such process, when M and D are in P, all other notations of such triplet have actually ceased to exist, and P is then exactly equal M or D of the circle, notwithstanding P cannot be definitely expressed short of its third power.

By such process, if the quadrant or diameter of the circle is definitely expressed in its first power, the other may be definitely expressed in its third power; consequently, when the diameter of the sphere is definitely expressed in its first power, its surface may be definitely expressed in its third power.

But as the third power of the bulk of the sphere whose diameter is 1, is expressed by the revolving decimal ,148148 *in infinitum*, hence when the diameter of a sphere is definitely expressed in its first power, its bulk cannot be definitely expressed in any

power; but the cube of its bulk will extend in a revolving but never ending decimal.

Nevertheless, the whole subject is properly referred to the method by way of rational analysis, which depends upon the great fundamental principle in polygonal economy, viz. that the product of the quadrant by the diameter of any given polygon, gives the area; and consequently, that area flows by a duplicate ratio to its linear measures; and consequently, that the diameter and quadrant have much to do in determining the ratio of one to the other.

As M is always conceived to be above N, however near the approach, such is therefore conceived to be the case in the final triplet, or where N. has ceased to exist, or rather, where there is no assignable difference between M and N; and as the value of x is then reduced to less than an assignable value, the difference between M and N of the final triplet is necessarily conceived to be proportionally the same as between M and B.

The antiquity, however, of Legendre's process, might be pleaded in its favor, were it not for the fact that its correctness has ever been doubted from its origin; for we have reason to believe that Archimides, even to the day of his death, had doubts in regard to the correctness of his exposition, and therefore sought to detect and relieve from the apprehended error, as thousands have since done and continued to do, until doubts were at length measurably relieved from by despair.

CHAPTER SECOND.

AN INQUIRY INTO THE *Modus Operandi* OF SOLAR ATTRACTION, AND OF THE SUSTAINING CAUSE OF THE SOLAR SYSTEM.

Perhaps any *theory* worthy the name, consists in a rational assignment of the causes of observed phenomena; and as has been well said, "No more causes are to be admitted than are sufficient to explain the phenomena;" for by so doing we shall be more likely to assign the true cause, or be less liable to account for phenomena in an irrational manner.

That like effects or phenomena are produced from like causes, is a necessary hypothesis in our enquiries; as by its adoption we acknowledge the immutability of the laws of nature, the evidence of which has hitherto been so fully sustained by observation and experiment; hence analogies in nature are to be received as legitimate evidence.

Any attempt to assign causes to phenomena which occur without the solar system, is perhaps too hypothetical for well authenticated philosophy; and perhaps true science will never require of us to account for the manner in which the planets and their satellites were created, or how they were first put in motion; but rather that we should take the solar system as we find it, and endeavor to account for the manner by which it is upheld and sustained.

Since the days of Kepler, the sustaining cause of the solar system has been ascribed to *attraction*, which like a messenger of omnipresence, continually acts upon and influences the revolving bodies, thereby causing them to tend towards the centre of the body

around which they immediately revolve, and by which they are retained in their respective orbits.

Kepler's first great law of planetary motion, viz: "That the squares of the periods of the planets are as the cubes of their mean distances from the sun," is full of instruction; as thereby we are enabled to compute their relative distances from the sun, and also their relative bulks or magnitudes compared with each other; as also the relative powers of attraction of the primary planets having satellites, in proportion to the bulks of those planets; by which we ascertain the fact, that the relative powers of attraction of the primary planets for their respective satellites in proportion to their bulks, are proportioned to each other in the inverse ratio of their relative distances from the sun: that is, their relative bulks multiplied by the inverse of their relative distances from the sun, gives their relative powers of attraction. Let us then seek for the cause why the attractive powers of the planets in proportion to their bulks or magnitudes, vary inversely as their distances vary from the sun; and endeavor to ascertain from analogies in nature or otherwise, whether the cause may not be rationally referred to some general law of nature coextensive with the universe, which may be mathematically assigned and computed,—as in case of Kepler's great law of planetary motions, viz: that the squares of the periods of the planets are as the cubes of their mean distances from the sun.

Observation and experiment, in the absence of all evidence to the contrary, have pretty fully established the important fact, that all bodies of matter are inert; that they only act as they are acted upon; and that they always act as they are acted upon.

Thus if a ball when put in motion by sudden impulse shall strike against another ball, the first ball acts upon the other as it is acted upon, and in no other manner.

If two bodies of matter be pressed against each other by continued force, each body acts as it is acted upon, and in no other way.

If a piece of soft iron be placed near a permanent magnet, the soft iron becomes a magnet, and remains so while thus situated, in which case it will attract iron filings; and in such case, the soft

iron acts upon the iron filings as it is acted upon by the permanent magnet, and in no other way or manner.

And if this ever mysterious operation of the law of nature, disclosed in magnetism, which we may contemplate whenever we please, should be the most happy illustration and perhaps the only perfect analogy of the *modus operandi* by which the solar system is sustained, I certainly would not exchange the evidence thus furnished, however natural and simple, for any mere hypothesis unsustained by evidence, that can be devised by man.

I shall therefore assume such to be the way and manner in which the solar system is sustained; and will denominate the same the *new theory*, in contradistinction to the theory which assumes that all bodies of matter possess an innate and independent power of attraction, and that such power is always the same in proportion to the quantity of matter contained in the body, wherever situated; which is here called the *old theory*.

I shall assume the *law* of gravity or attraction to be thus:

The force of gravity or attraction varies inversely as the distance varies from a centre of gravity; which *law* I will familiarly call the *new law*, in contradistinction to the hypothesis which assumes that the force of gravity varies inversely as the square of the distance varies from a center of gravity; which law I shall denominate *the old law*. And it is my purpose to compare the *new* theory and law with the *old*, with a view to disclose some of the results and consequences to which they must necessarily give rise.

In making such comparison, I will select for consideration that important phenomenon in the solar system which has given rise to the hypothesis of the *different densities* of the planets; which phenomenon, from its uniformity throughout the solar system, would seem entitled to the assignment of some general law as its cause; the phenomenon to which I refer being no less than that of the different attractive powers of the primary planets in proportion to their bulks or magnitudes, which is found from observation to be in the inverse ratio of their distances from the sun.

For although at first, the phenomenon was only found to apply to the planets having satellites, it was subsequently found by Eu-

ler, La Grange, and other eminent mathematicians, to apply as well to the other planets as to those having satellites, and accordingly adopted the phenomenon as a general rule by which to compute the disturbing forces of the planets and their consequent perturbations.

And this important phenomenon in the solar system, in lieu of being referred to some general law coextensive with the law of gravity, as its source,—has been assigned to mere chance, viz : to supposed different densities of the planets,—the truth of which we know nothing ; nor have we any means of assigning the cause, even though the hypothesis were found to be true.

And although the computation of the supposed different densities of the planets has sometimes been esteemed as a kind of intellectual triumph, it is yet possible that the hypothesis itself, in regard to the different densities of the planets, is but the natural offspring of a previous error : As by this hypothesis alone, a reconciliation could be had between the *old theory and the law* of gravity in regard to the different attractive powers of the planets in proportion to their bulks or magnitudes : For the *old theory and law* of gravity have no direct communion with each other in sustaining or governing the solar system.

And it may yet become a wonder why the old theory should have been adopted without a title of evidence by way of analogy or otherwise, to support it, unless it were for the express purpose of accounting for the different attractive powers in proportion to their bulks, upon the hypothesis of different densities,—in as much as the old law has nothing whatever to do with the subject.

Let us compare the new theory with the old, by way of example.

Suppose an assumed planet of equal bulk or magnitude with the earth, to revolve at four times the mean distance from the sun that the earth does; and suppose such planet to have a satellite revolving at the same distance from the planet that the moon does from the earth.

Now observation has induced a general rule which from its uniformity has been resolved into a general law which is found to never fail when it is applied, viz : that the squares of the periods

of the planets are always found to be as the cubes of their mean distances from the sun; and consequently that the motion of the earth in its orbit is twice as great as the motion of the planet revolving at four times the distance of the earth from the sun.

So also observation has induced a rule equally uniform and conclusive, by which we determine the fact that the motion of the moon in its orbit is twice as great as that of the satellite of the other planet; and that we as readily compute the relative motions of the satellites of different planets, as of the planets themselves; for notwithstanding, for a time, the rule was thought not to apply to the satellites of Herschel, nevertheless, subsequent observations have proved that Herschel and its satellites fall within the same general rule.

Now the old law renders no aid whatever in accounting for the observed phenomenon, that the attractive powers of the primary planets, in proportion to their bulks, vary in the inverse ratio of their distances from the sun. Hence, whilst we retain the old law, the observed phenomenon must be accounted for by theories and hypotheses wholly independent of the law: Hence, upon the theory of innate attraction in proportion to the quantity of matter, wherever situated,—in connection with the hypothesis of the different densities of the planets, rests the whole cause of the different attractive powers of the planets in proportion to their bulks.

And it would seem, perhaps, too empyric, to assign as the cause of one of the most important phenomena in the solar system to two naked, distinct, and disconnected hypotheses, neither of which, perhaps, is sustained by a single analogy or similitude in nature, when the same may be perfectly accounted for by universal laws which we know to exist, and which may be demonstratively proved to exist.

Nor has the old law, as yet, rendered any aid whatever, in accounting for the motions of the planets or of their satellites; no, not even since the days of Clairaut, who, finding, with other mathematicians of Europe, that but one half of the motion of the moon's apogee was accounted for by the force of gravity as the law then stood, supposed that he had corrected the evil by adding or increasing the amount of force to just twice the amount of force

given by the old law; which amount so added, had it not been wholly factitious, unavailable, and even non-existent, would have increased the amount of force to just one half the required amount; for in case of a revolving body, twice the motion always requires four times the amount of force; And the result of Clairaut's amendment of the law, is perhaps found in the constant multiplication of empyric equations of the moon from that time to the present.

Now by the new theory and law, we have no occasion for the hypothesis of the different densities of the planets; thus as the earth and the assumed planet are of equal magnitudes, we naturally conclude that their quantities of matter will not materially differ, if no good evidence is shown to the contrary.

If, therefore, in accordance with the new theory, both planets receive their attractive powers from the sun, and become attractive in the same proportion that they are attracted; and if in accordance with the new law, the earth is attracted by four times the force that the other planet is,—and therefore exerts four times the force upon its satellite that the other planet does, it is manifest that the motion of the moon must be twice as great as the motion of the other satellite, in order to retain it in its orbit; hence, by the new theory and law, the densities of the planets would not sensibly differ.

And if there be any dependence whatever to be placed upon the principle supposed to be so well established, viz: that twice the motion of a revolving body requires four times the force of attraction to retain the body in its orbit, then the new law by which the earth is attracted by four times the force of the other planet, and by which both its *gravity* and *ponderosity* is four-fold that of the other planet, must surely prevail in regard to the two planets, unless *distance* compensates for, and dispenses with the *force of gravity* just in proportion as the distance is increased; which is one of the prominent features of the old law, but from what evidence or authority we are not informed; For in regard to the two planets, by the old law, sixteen times the force is required to balance twice the motion.

As the *gravity* of both planets depends upon the force of attrac-

tion exerted by the sun upon the planets, hence the gravity of the planets is as the force of gravity or attraction, and consequently varies in some given inverse ratio to the distance; And by the old law, the gravity of the earth is sixteen times as great as the gravity of the other planet; but by the new law it is only four times as great.

If the *ponderosity* of a planet is equal to the force or power required to sustain the planet in a state of rest, and prevent its falling towards the sun,—then the product of the quantity of matter contained in a planet,—by its gravity, gives its ponderosity: hence by the old law, the ponderosity of the earth is sixty-four times as great as of the other planet, but by the new law, it is four times as great; that is, by the new law, the gravity of either is equal to its ponderosity, and the attractive power of either is equal to its gravity or ponderosity; whilst by the old law, the attractive power of the earth, (whose distance, gravity or ponderosity, is properly assumed at unity,) is equal to its gravity or ponderosity; whilst the attractive power of the other planet, (whose distance is four units,) is four times as great as its gravity, and sixteen times as great as its ponderosity.

Suppose the central force situated in the sun to be increased four fold: In such case, the motions of the two planets must be doubled in order to retain them in their orbits: But by the old theory, as well as by the hypothesis of the different densities of the planets, such increased power of the sun, and the consequent increased motion of the planets, would furnish no cause whatever for an increase of motion in the satellites, as by the old theory it would not increase the attractive power of the planets; the attractive power of the planets, by the old theory, depending wholly upon the quantity of matter contained in the respective planets. Whereas by the new theory, the attractive power of the planets would increase with the increase of the central power.

But I will proceed to a consideration of the *law of gravity*,—believing that a true assignment of the law will serve to remedy all errors in the theory of gravity.

OF THE LAW OF SOLAR ATTRACTION—OTHERWISE CALLED THE LAW OF GRAVITY.

The *law of gravity* is here used to imply the inverse ratio by which the force of attraction varies as the distance varies from a center of attraction or gravity; as in case of a body falling from a state of rest towards the earth at different distances from its center, or of planets revolving at different distances from the sun.

The question here raised in regard to the *law of gravity* is, whether the force of gravity varies inversely as the distance, or inversely as the square of the distance varies from a center of gravity; the first hypothesis being familiarly called *the new law*, and the second hypothesis, *the old law;* and it cannot be expected that evidence can be adduced from the economy of nature to sustain both hypotheses.

Colin Maclaurin, who was an early and conspicuous commentator upon the law of gravity, and whom alone I shall have occasion to quote, for the reason that others have merely adopted and endorsed his exposition of the law, without adding one ray of light thereto, notwithstanding he adopted and promulgated the old law, was nevertheless constrained to insist that such is not the law by which the planets can best revolve; and suggests that the proper limit of gravitation by which the planets could best revolve, is the inverse simple ratio of the distance; as also how admirably they would revolve from one apsis to the other, were such the law.

In an investigation of the law of gravity, our evidence, and consequently the premises from which we make our deductions or inferences, are perhaps to be generally drawn from the common concerns of life—from the law of falling bodies—and from Kepler's laws of planetary motion.

In referring to the common concerns of life, I will suggest the case of the husbandman, who has a specific operation, or amount of labor to be performed which one man can perform in two days, and which two men can perform in one day; And also to the mechanic, who would raise a weight or ponderous body over a given amount of space by means of the wheel and axle, which

two men can perform in one hour, and which one man can perform in two hours.

Now in those operations, we have the following principles disclosed in regard to the action or operation of force in time :

First—The amount of force expended in the performance of a given operation is the same, whatever be the *rate* of force applied or whatever be the time spent in the operation.

Second—The *whole time* spent in the performance of the given operation is the reciprocal of the *rate* of force applied ; and consequently, the whole time spent in the performance of the given operation varies inversely as the rate of force is applied.

Third—If the specific operation be assumed at unity,—that is, if the amount of force required to be expended in the specific operation be assumed at unity, as that of the labor of two men for example ; and the whole time of performing the operation be assumed at one unit of time, as one day, for example, the *rate* of force applied is consequently assumed at unity, by which to compare the rate of force when the whole time in which the amount of force is expended, shall be more or less than one unit : Hence if the whole time in which the given operation is performed, or unit of force is expended, be two or more units, the *amount* of force expended in either unit of time is equal to the *rate* of force applied.

Fourth—The *rate* of force varies as the *amount* of force expended in a given time varies,—or *vice versa.* The amount of force expended in a given time varies as the rate of force applied: Hence if the rate of force applied for a given time be doubled, the amount of force expended in the given time is doubled : Or if the amount of force expended in a given time is doubled, it is *because* the rate of force applied for the same time is doubled.

Nor is it material from what cause, or in what way or manner the *rate* of force shall be increased or decreased, in as much as the *whole time* in which the specific operation is performed, varies inversely as the *rate* of force applied varies.

Hence in case of a body falling from a state of rest at different distances from the center of the earth, as at one unit of distance, and again at two units of distance, if we conceive the rate of force,

or force of gravity, to be the same at one distance as at the other, we necessarily conceive the same *amount* of force to be expended, and the same amount of space to be passed over or described in a given amount of time, as in one second of time, or in one minute of time, for example, in one operation as in the other.

Hence the *amount* of force expended in the fall of the body from a state of rest over a given or specific amount of space, is the same at one distance from the center of the earth as at another; but as the rate of force applied to the body has an inverse increase or decrease to that of distance from the center of the earth or center of gravity, hence as the distance is greater, the whole time spent in the specific operation is greater; and the whole time spent in the specific operation varies inversely as the *rate* of force is caused to vary by reason of different distances from the center of the earth.

Hence, in case of the body falling from a state of rest over equal amounts of space, at two different distances from the center of the earth, if we can ascertain the proportional amount of time spent in the two operations, with the distances from the center of the earth at which the operations occur, we consequently determine the inverse ratio of the *rate* of force to the distance.

Mr. Maclaurin has properly adopted unity or 1, as the numerical standard measure of all the elements used in our computations; that is, when the distance is unity, the *time* in which the specific operation is performed is assumed at unity,—and the *rate* of force is then assumed at unity, by which to proportion the *time* and *rate* of force when the distance is more than one unit; hence all magnitudes of the elements are considered as numerical magnitudes; and when two or more elements are said to be equal, or proportional, it implies numerical equality or proportion.

Mr. Maclaurin has properly assumed the distance from the center of the earth to its surface, at *one unit of distance;* and the mean distance of the moon at sixty units of distance. He assumes one second of time as one unit of time; and for the reason that it has been found from frequent experiments that a body falling from a state of rest at the surface of the earth, or at one unit of distance, will fall over or describe 15 1-12 Parisian feet in one second of time, he has assumed 15 1-12 Parisian feet as one unit of space;

and *the first unit of space* described by the body on falling from a state of rest at any given distance from the center of the earth, is called the *specific operation*, however far the operation may be extended beyond the first unit of space.

Mr. Maclaurin consequently assumes the *rate* of force or gravity at unity, at the surface of the earth, or at one unit of distance, or when the first unit of space is described in one unit of time; and also that one unit of force is expended in the specific operation, at the surface of the earth; for such is necessarily the case upon any hypothesis in regard to the law of gravity.

Mr. Maclaurin properly assumes that the specific operation will be performed in sixty units of time at the distance of the moon, or in one minute of time; that 3,600 units of space will be described in 60 units of time, or in one minute of time at the surface of the earth, and that 3,600 units of space will be described in one hour at the distance of the moon.

Mr. Maclaurin properly assumes, (in accordance with the *law* of falling bodies, which is the same at all distances,) that 3,600 times the space will be described in 60 units of time from the beginning of the fall that is described in the first unit of time; and consequently, that 3,600 times the space is described in the first unit of time at the surface of the earth, that is described in the first unit of time at the distance of the moon.

Mr. Maclaurin also properly assumes that at any given distance, the *rate* of force applied is uniform throughout the operation; and consequently, that during any given operation, equal amounts of force are expended in equal times; that is, during any given operation, the same amount of force is expended in one unit of time as in another.

Hence Mr. Maclaurin necessarily assumes, that in any given operation, viz., at any given distance, 60 fold the time spent in the description of 3,600 units of space that is spent in the description of the first unit of space; and 60 fold the amount of force is expended in the description of 3,600 units of space that is expended in the description of the first unit of space.

He therefore necessarily assumes as an unavoidable consequence, that the same *amount* of force is expended in one second of time at

the surface of the earth that is expended in one minute at the distance of the moon; or that there is the same amount of force expended in one minute at the surface of the earth as in one hour at the distance of the moon.

That the force expended in the first unit or second of time at the surface of the earth describes one unit of space,—and the force expended in sixty units of time, or in one minute, describes 3,600 units of space.

That the force expended in the first unit or second of time at the distance of the moon describes 1-3600 part of the first unit of space; the force expended in 60 units of time or in one minute, at the distance of the moon describes the first unit of space; and the force expended in one hour at the distance of the moon describes 3,600 units of space.

Hence we compel Mr. Maclaurin to assume, that force is expended 60 times as fast, (and only 60 times as fast,) at the surface of the earth as at the distance of the moon,—whilst space is described 3,600 times as fast at the surface of the earth as at the distance of the moon. And if force is expended 60 times as fast, it is simply for the reason that the *rate* of force applied is 60 times as great.

Hence if the *rate* of force applied is 60 times as great, the amount of space described in a given time will be 3,600 times as great; and such would be the case either at the surface of the earth, or at the distance of the moon.

Now it will be readily seen on careful examination, that I have not caused Mr. Maclaurin to assume a single position or principle that is not directly incident to the *law* of falling bodies; Nevertheless, from those premises, Mr. Maclaurin infers that the force of gravity varies inversely as the *square* of the distance varies; But from which I undoubtingly infer, that the force of gravity varies inversely as the distance varies.

Mr. Maclaurin in deducing inferences from premises, says, "The computation may be made in this manner,—the mean distance of the moon from the earth being 60 times the distance of heavy bodies at its surface from its center, (a body at the surface will fall 15 1-12 Parisian feet in one second,) and her gravity increasing

in proportion as the square of the distance from the center of the earth decreases, her gravity would be 60 X 60 times greater near the surface of the earth than at her mean distance, and therefore would carry her through 60 X 60 15 1-12 Parisian feet in a minute near the surface; but the same power would carry her through 60 X 60 times less space in a second than in a minute, by what has been often observed of the descent of falling bodies; and therefore the moon in a second of time would fall by her gravity near the surface of the earth 15 1-12 Parisian feet, which therefore is the same with the gravity of terrestrial bodies."

Now the allegations and assumptions contained in the foregoing exposition of Mr. Maclaurin, which are in accordance with the law of falling bodies, are those which assume that the moon, or any other body, on falling from a state of rest at the surface of the earth, will describe the first unit of space in one unit of time—and 3,600 units of space in 60 units of time: And that the moon or any other body on falling from a state of rest at the distance of the moon, will describe the first unit of space in 60 units of time.

And the allegations and assumptions contained in Mr. Maclaurin's said exposition which are not in accordance with the law of falling bodies, are those which assume, that if the moon (or any other body at the distance of the moon,) were brought nearer the earth, its gravity would increase in proportion as the square of the distance decreased; thereby making the gravity of the moon 3,600 times greater at the surface of the earth than at her mean distance; And also the very extraordinary deduction drawn from such assumption, viz. that it would in consequence, describe only 3,600 units of space in one minute, or in 60 units of time at the surface of the earth.

Whereas we have compelled Mr. Maclaurin to assume in the premises, not only in accordance with all we know in regard to the law of falling bodies,—but also in accordance with all we know in regard to the action or operation of force in time, that if the force of gravity or rate of force were 3,600 times as great at the surface of the earth as at the distance of the moon,—that if the body on falling from a state of rest at the distance of the moon would describe the first unit of space in 60 units of time, it would, on

falling from a state of rest at the surface of the earth, describe 3,600 units of space in one unit, viz. in one second of time.

That is,—if the *rate* of force, (and consequently the gravity of the body,) be 3,600 times as great at the surface of the earth as at the distance of the moon, the body on falling from a state of rest at the surface of the earth would describe space near thirteen millions of times as fast as the body would on falling from a state of rest at the distance of the moon; whereas it in fact, describes space only 3,600 times as fast; and consequently, its gravity at the surface of the earth is only 60 times as great as at the distance of the moon.

Mr. Maclaurin's error, therefore, lies firstly, in his naked and *a priori* assumption, that the gravity of the moon on being brought nearer the earth would be increased in proportion as the square of the distance should decrease; whereas the ratio by which the gravity of the moon would increase as the distance should decrease, is the only question we are wishing to solve, and should be the result of a deduction drawn from well advised premises, rather than depend upon naked hypothesis, lest we lose all benefit whatever, to be derived from premises well considered.

And his second error consists in his unaccountable inference drawn from his premises,—viz. that the moon thus increased in gravity, on falling from a state of rest at the surface of the earth, would describe only 3,600 units of space in 60 units of time,—or one unit of space in the first unit of time.

Hence the reason why the moon on falling from a state of rest at the surface of the earth will describe the first unit of space in one unit of time,—and on falling from a state of rest at the distance of the moon, will describe the first unit of space in 60 units of time, is not *because* the moon on being brought nearer the earth will increase in gravity in the same proportion as the *square* of the distance decreases; but it is because the moon on being brought nearer the earth will increase in gravity in proportion as the distance decreases.

By the well known law of falling bodies, which is the same at all distances from the center of the earth, at any given distance the amount of space described in the first unit of time is one

third as great as the amount described in the second unit of time,—one fifth as great as the amount of space described in the third unit of time, and so on,—the amount of space described in the successive units of time increasing in value as the odd numbers in their order increase,—thus—1—3—5—7 and so on; and at any given distance, the amount of space described in any unit of time during the operation, may be denominated an *increment* of space.

Hence when the distance at which the fall begins is one unit, the first increment is equal to the first unit of space; when the distance is two units, the first unit of space consists of two increments,—and the first increment is but one fourth of the first unit of space; when the distance is three units, the first unit of space consists of three increments—and the first increment is but one ninth of the first unit of spaee, and so on—until when the distance is sixty units, the first unit of space consists of sixty increments,—and the first increment is but 1-3600 part of the first unit of space.

It is therefore manifest that space is described at the surface of the earth 3,600 times as fast as at the distance of the moon.

In any given operation, the amounts of force expended on the several increments of space, are equal, one with another: Hence in any given operation, the amount of space described, constantly increases as the square of the amount of force expended increases.

If the fall commences at the surface of the earth, then, however far the operation may be continued, one unit of force is expended on one increment of space,—and consequently, at any instant of time during the operation, the amount of space described is the square of the amount of force expended.

And such is as necessarily the case at any other distance as at the surface of the earth; for any other distance may as well be assumed at one unit of distance as at the surface of the earth,—as the distance of the moon for example, in which case if 15 1-12 Parisian feet were still retained as the unit of space, then one minute would be one unit of time.

Hence the principle is general—viz. at whatever distance the fall may commence—at any stage of the operation, however far

continued, the amount of space described is the square of the amount of force expended in its description : Nor can such be the case otherwise than upon the principle that the *rate* of force varies inversely as the distance varies ; and consequently, that the same amount of force is expended on the first unit of space, or in the specific operation, at one distance as at another : The distance having no other effect in the operation than that of varying the *rate* of force ; the time spent in the operation being the reciprocal of the rate of force.

As the *rate* of force applied is always necessarily increased as the amount of force expended in a given time is increased,—or as the amount of force expended in a given time is increased as the rate of force applied for the given time increased,—hence the rate of force in descending from the distance of the moon to the surface of the earth is increased in the same proportion that the amount of force expended in one unit of time at the surface of the earth, for example, exceeds the amount of force expended in one unit of time at the distance of the moon,—which amount of force expended in the first unit of time at the surface of the earth is sixty times greater than at the distance of the moon, because it will describe 3,600 times the space : And in such case, the whole office of distance consists in varying the rate of force.

Hence the rate of force at the surface of the earth must be 60 times as great as at the distance of the moon, in order that the amount of force expended in a given time at the surface of the earth may be 60 times as great as at the distance of the moon.

Finally,—the gravity of the body, or rate of force, varies inversely as the distance varies, for the reason that the time is as the distance, or varies with the distance ; That is, the number of units of time spent in the specific operation, viz. in the description of the first unit of space, is always equal to the number of units in the distance at which the fall commences.

Thus we proceed upon the principle that when the distance is one unit, the time spent in the specific operation is one unit ; when the distance is two units, the time spent in the specific operation is two units, and so on.

And as the whole time spent in a specific operation always

varies inversely as the rate of force applied varies, hence, in case of the falling body, as the time is as the distance, or varies with the distance, hence the distance varies inversely as the rate of force varies.

Now some few of the consequences that would result from Mr. Maclaurin's exposition, are the following, viz :

The gravity of the body would be 3,600 times as great at the surface of the earth as at the distance of the moon.

The amount of force expended in one second of time at the surface of the earth would be equal to the amount expended in one hour at the distance of the moon.

Sixty fold the amount of force would be expended in the description of the first unit of space at the surface of the earth as at the distance of the moon.

Amount of force would be dispensed with in the description of the first unit of space, just in proportion as the whole time spent in its description exceeds unity, or one unit of time ; and *rate* of force would be dispensed with in proportion as the square of the whole time exceeds unity ; thereby making the passive element of time, the principal operator in the description of space.

The time spent in the specific operation would vary inversely as the amount of force expended in the operation varies ; and the *square* of the whole time spent in the specific operation would vary as the rate of force varies.

Such, then, are some few of the consequences following from Mr. Maclaurin's exposition of the law of falling bodies.

Nevertheless, the time spent in the specific operation at any given distance, is the reciprocal of the rate of force applied ; and if the time spent in the specific operation is greater at one distance than at another, it is because the *rate* of force is proportionally less : If 60 fold the time be spent (the *rate* of force remaining the same,) 60 fold the *amount* of force will be expended—and 3,600 fold the space will be described : And if the *rate* of force be increased 60 fold, (the time remaining the same,) 60 fold the *amount* of force will be expended—and 3,600 fold the space will be described.

Note—Upon any hypothesis in regard to the law of gravity,

there is no difficulty in proportioning the elements of time, distance, space and motion to each other.

In case of the falling body, the *rate* of motion is properly assumed at unity when the distance is one unit: And as the motion is properly referred to space described, the motion acquired in the description of the first unit of space, is the proper standard by which to compute and proportion the *rate* of motion: Hence upon any hypothesis in regard to the law of gravity, the rate of motion will vary inversely as the distance varies, and consequently, inversely as the whole time varies in which the first unit of space is described.

Thus when the first unit of space is described in two units of time, the rate of motion is but half as great as when the first unit of space is described in one unit of time.

In any given operation of the falling body, the motion acquired in the first unit of time, is numerically equal to the first increment of space, or space described in the first unit of time; consequently the motion acquired in the first unit of time varies inversely as the square of the distance varies; whilst the motion acquired in the first unit of space varies inversely as the distance varies.

The motion acquired in the description of the first unit of space at the surface of the earth is 60 times as great as the motion acquired in the description of the first unit of space at the distance of the moon: Hence the motion acquired in one second of time at the surface of the earth is equal to the motion acquired in one hour, or in 3,600 units of time at the distance of the moon.

OF THE LAW OF GRAVITY IN CONFORMITY WITH KEPLER'S TWO LAWS OF PLANETARY MOTION.

First—That the squares of the periods of the several planets of a system are as the cubes of their *mean* distances from the sun or central force—whether the planets revolve in centric orbits, or in eccentric orbits—or some of them in centric, and others in eccentric orbits.

Secondly—That the motion of a planet revolving in an eccentric orbit, varies inversely as the distance of the planet shall vary in its revolution round the sun.

A planet revolving in an eccentric orbit, has its mean distance—its mean motion—and its mean force of gravity; and the mean motion and mean force of a planet are its motion and force at its mean distance.

The *mean* distance of a planet from the sun is, by any system, half the distance from the perihelion to aphelion; for such is the mean distance implied in Kepler's first law: Hence neither the mean distance, mean force, or mean motion, is a mean proportional between the perihelion and aphelion distance, force or motion: And the *mean distance* is here used wholly in conformity with Kepler's first law; and has nothing to do with the form of the orbit otherwise than that the orbit is *eccentric.*

The mean distance—mean motion—and mean force of a planet are proportioned to each other the same that they would be, were the planet to revolve in a centric orbit at such mean distance; Hence upon any hypothesis in regard to the law of gravity, the mean motion of the planet is the reciprocal of the square root of the mean distance; for such is always the case with a planet revolving in a centric orbit: Hence we have no difficulty in proportioning the period—the mean distance—and the mean motion, to each other.

In proportioning the elements of a planet revolving in an eccentric orbit to each other, it is necessary to give its mean distance, and its perihelion distance.

In a comparison of the elements of two or more planets re-

volving at different mean distances from the sun, each element of one of the planets is properly assumed at unity, by which to proportion the elements of the other planet, viz., its period,—the length of its orbit,—the amount of force expended in the period,—the mean distance,—the mean force,—and mean motion: And when the mean distances of all the planets are proportioned to that of the planet whose mean distance is assumed at unity,—the square of the period of either planet of the system is the cube of its mean distance from the sun.

Hence, in comparing and proportioning the elements of any individual planet with each other, we assume its mean distance at such numerical value as we please, as at one or more units: And if the orbit be eccentric, we also give its perihelion distance.

Let a planet revolve in an orbit whose mean distance is four units, and whose perihelion distance is one unit,—in which case its period will be eight units,—and its mean motion will be half of unity.

Now in such orbit, the motion and the force decrease while the distance increases,—and increase while the distance decreases; and by Kepler's second law, the motion constantly varies inversely as the distance varies.

By any hypothesis in regard to the law of gravity, the perihelion motion of the planet is two units,—its mean motion is half of unity,—and its aphelion motion is 2·7 of unity; and by any hypothesis, the perihelion force of the planet is unity.

If the force varies inversely as the distance varies, the perihelion force is one unit—the mean force is half of unity,—and the aphelion force is 1·7 of unity; but if the force varies inversely as the square of the distance varies, the perihelion force is one unit,—the mean force is 1·16 of unity,—and the aphelion force is 1·49 of unity.

If the force varies inversely as the distance varies, the force and motion preserve the same proportions to each other throughout the orbit, that they have at the mean distance;

and such will be the case in any eccentric orbit, whatever be its mean and perihelion distances; that is,—the motion increases or decreases in the same ratio that the force does; and either of them increase or decrease in the inverse ratio of the distance.

If therefore the force of gravity varies inversely as the distance varies, the mean force of a planet is always the square of the mean motion; the force of gravity or *rate* of force is always the reciprocal of the distance at all points of the orbit; the force of gravity varies as the motion varies, and either of them varies inversely as the distance of the planet varies in its orbit; the amount of force expended on the planet during its period is the cube root of the period, or is the reciprocal of the mean motion; and the space passed over or described by the planet during the period, is the square of the amount of force expended in the period.

But if the force of gravity varies inversely as the square of the distance varies, then—the mean force of a planet is always the fourth power of its mean motion; the force of gravity or rate of force, is the reciprocal of the square of the distance at all points of the orbit; the amount of force expended on the planet during its period, is equal to the mean motion, and consequently the amount of force expended on a planet whose mean distance is four units, and its period is eight units, would be but half as great as the amount of force expended on a planet whose mean distance is one unit, and whose period is one unit.

But let us refer to the acknowledged principle in regard to planetary force and motion, viz: That twice the motion of a planet revolving in a centric orbit, for example, requires quadruple the force to retain the planet in its orbit: Or if twice the mean motion be given to a planet, the mean force must be increased four-fold in order to retain the planet in its orbit.

That is, so often as the mean motion is doubled, so often the mean force must be increased four-fold: So often as the mean motion shall be increased three-fold, so often the mean force must

be increased nine-fold, and so on, in order to retain the planet in its orbit.

Hence the only inference that can be drawn from this principle is, That at any possible distance at which a planet may revolve, the mean force is the square of the mean motion,—and not the fourth power of the mean motion.

The hypothesis, therefore, which assumes that the force of gravity varies inversely as the square of the distance varies, makes distance an operator to the exclusion of force, in retaining a planet in its orbit, just in proportion as the mean distance is increased above unity.

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